$y = f(x)$ from $x = a$ to $x = b$, revolved around the x-axis:

$$S = 2\pi\int_a^b f(x)\underbrace{\sqrt{1+(f'(x))^2}\,dx}_{ds}.$$

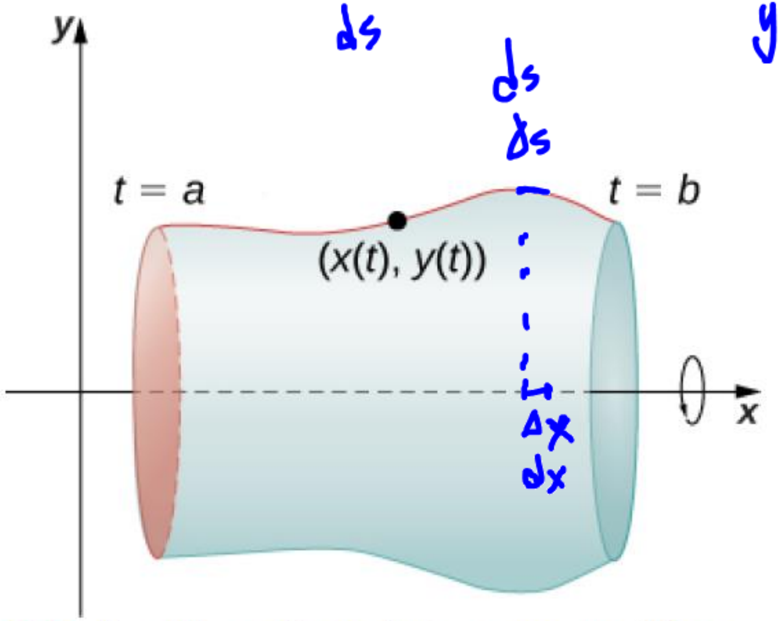
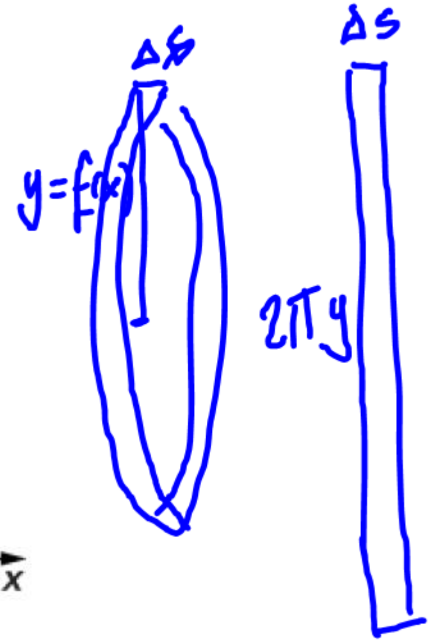


Figure 1.25 A surface of revolution generated by a parametrically defined curve.

$$S = 2\pi\int_a^b y(t)\underbrace{\sqrt{(x'(t))^2+(y'(t))^2}\,dt}_{\Delta s}$$



$\Delta A = 2\pi f(x)\,\Delta s$

Σ

$$\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

1.9 Find the surface area generated when the plane curve defined by the equations

$$x(t) = t^3, \quad y(t) = t^2, \quad 0 \le t \le 1$$

is revolved around the x-axis.

$x'(t) = 3t^2 \quad y'(t) = 2t$

$dt \cdot t^3 = t^2\, t\, dt$

$= \left(\frac{u-4}{9}\right) \frac{du}{18}$

$$2\pi \int_0^1 t^2 \sqrt{(3t^2)^2 + (2t)^2}\, dt = 2\pi \int_0^1 t^2 \sqrt{9t^4 + 4t^2}\, dt$$

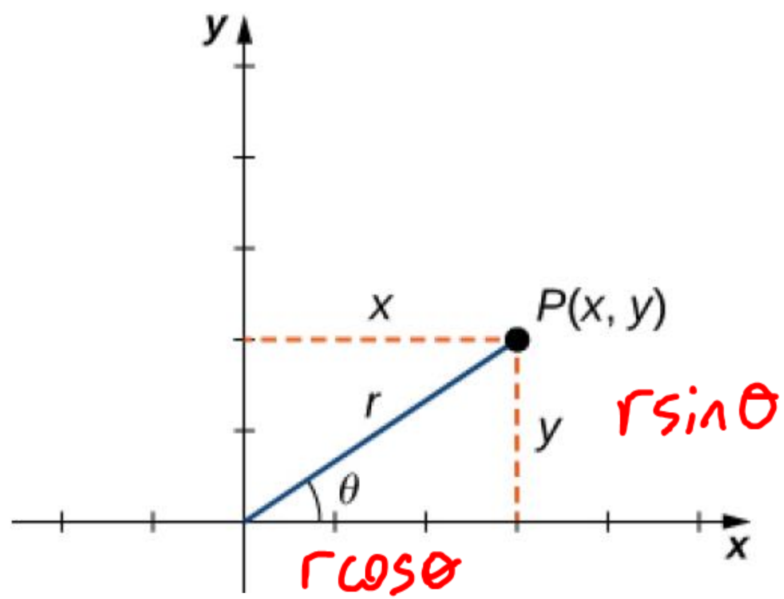
$$= 2\pi \int_0^1 t^3 \sqrt{9t^2+4}\, dt \qquad = 2\pi \int_4^{13} \left(\frac{u-4}{9}\right) u^{1/2} \frac{du}{18}$$

$u = 9t^2 + 4 \qquad t^2 = \frac{u-4}{9}$

$du = 18t\, dt$

$$= \frac{\pi}{81} \left(\frac{u^{5/2}}{5/2} - 4 \frac{u^{3/2}}{3/2} \right) \Bigg|_4^{13}$$

$$= \frac{\pi}{81} \left[\left(\frac{2}{5} 169\sqrt{13} - \frac{8}{3} 13\sqrt{13} \right) - \left(\frac{64}{5} - \frac{64}{3} \right) \right]$$



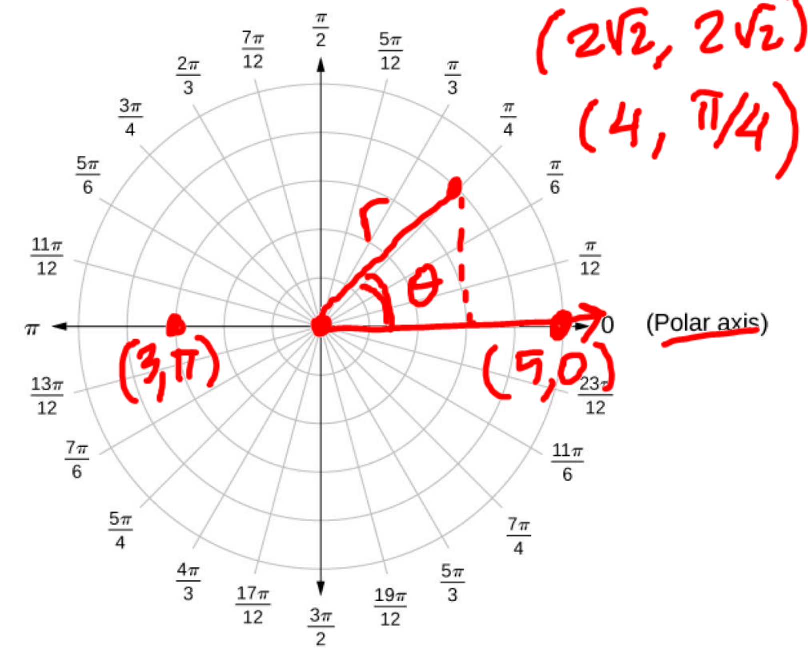


Figure 1.28 The polar coordinate system.

Theorem 1.4: Converting Points between Coordinate Systems

Given a point P in the plane with Cartesian coordinates (x, y) and polar coordinates (r, θ), the following conversion formulas hold true:

$$x = r\cos\theta \text{ and } y = r\sin\theta, \tag{1.7}$$

$$r^2 = x^2 + y^2 \text{ and } \tan\theta = \frac{y}{x}. \tag{1.8}$$

$\theta = \arctan\left(\frac{y}{x}\right)$

These formulas can be used to convert from rectangular to polar or from polar to rectangular coordinates.

1.10 Convert $(-8, -8)$ into polar coordinates and $\left(4, \frac{2\pi}{3}\right)$ into rectangular coordinates.

$r \quad \theta$ $\qquad$ (x, y) $\qquad$ $\frac{2\pi}{3} = 120^\circ$

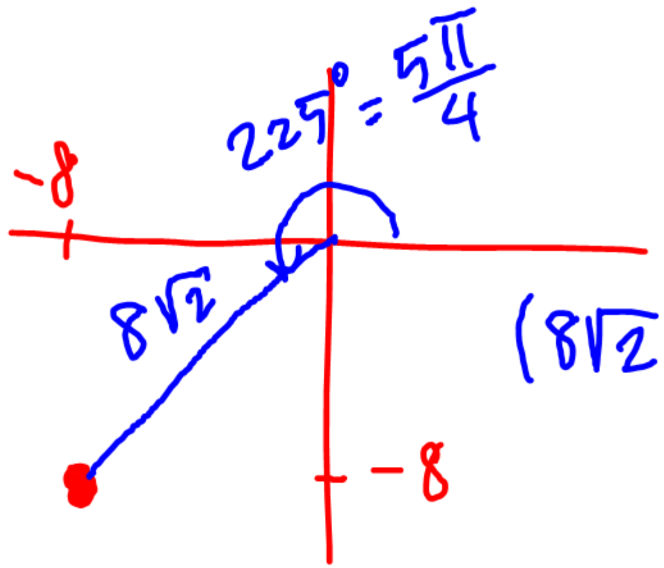
$$r = \sqrt{x^2 + y^2} = \sqrt{8^2 + 8^2} = 8\sqrt{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-8}{-8} = 1 \qquad \theta = \frac{\pi}{4}$$

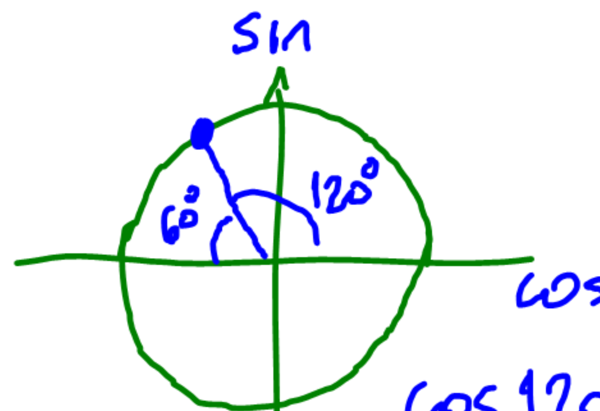
$$\theta = \frac{5\pi}{4}$$

$$x = r\cos\theta = 4\cos\frac{2\pi}{3} = 4\left(-\frac{1}{2}\right) = -2$$

$$y = r\sin\theta = 4\sin\frac{2\pi}{3} = 4\,\frac{\sqrt{3}}{2} = 2\sqrt{3}$$



$$\left(8\sqrt{2}, \frac{5\pi}{4}\right)$$



$$\cos 120^\circ = -\cos 60^\circ$$

$$\sin 120^\circ = \sin 60^\circ$$

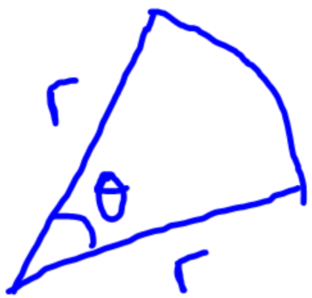
1.12 Create a graph of the curve defined by the function $r = 4 + 4\cos\theta$.

1.13 Rewrite the equation $r = \sec\theta\tan\theta$ in rectangular coordinates and identify its graph.

Theorem 1.6: Area of a Region Bounded by a Polar Curve

Suppose f is continuous and nonnegative on the interval $\alpha \le \theta \le \beta$ with $0 < \beta - \alpha \le 2\pi$. The area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta. \tag{1.9}$$



$$\Delta A = \frac{\Delta\theta}{2} r^2$$

$$A = \int_{\alpha}^{\beta} \frac{d\theta}{2} r^2$$

$$\frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\theta}{2} r^2$$

1.15 Find the area inside the cardioid defined by the equation $r = 1 - \cos\theta$.

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta = \frac{1}{2}\int_{0}^{2\pi} (1-\cos\theta)^2\,d\theta$$

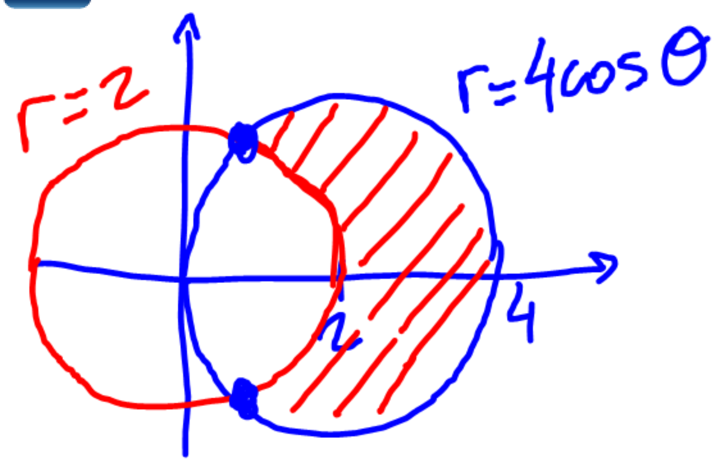
$$= \frac{1}{2}\int_{0}^{2\pi} \left(1 - 2\cos\theta + \cos^2\theta\right)d\theta = \frac{1}{2}\left[\theta - 2\sin\theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2}\right]\Bigg|_{0}^{2\pi}$$

$$\int \cos^2\theta\,d\theta = \int \left(\frac{\cos 2\theta + 1}{2}\right)d\theta = \frac{\sin 2\theta}{4} + \frac{\theta}{2} + c$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= \frac{1}{2}\left[3\pi - 0 + 0 - (0 - 0 + 0)\right] = \frac{3\pi}{2}$$

1.16 Find the area inside the circle $r = 4\cos\theta$ and outside the circle $r = 2$.



$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} \text{ OR } \frac{5\pi}{3}$$

$$A = \frac{1}{2}\int_{\pi/3}^{5\pi/3} \left((4\cos\theta)^2 - 2^2\right) d\theta$$

$$=$$

$$A = \frac{1}{2}\int_{\alpha}^{\beta} \left(r_{out}^2 - r_{in}^2\right) d\theta$$

$r = 2\sin\theta$

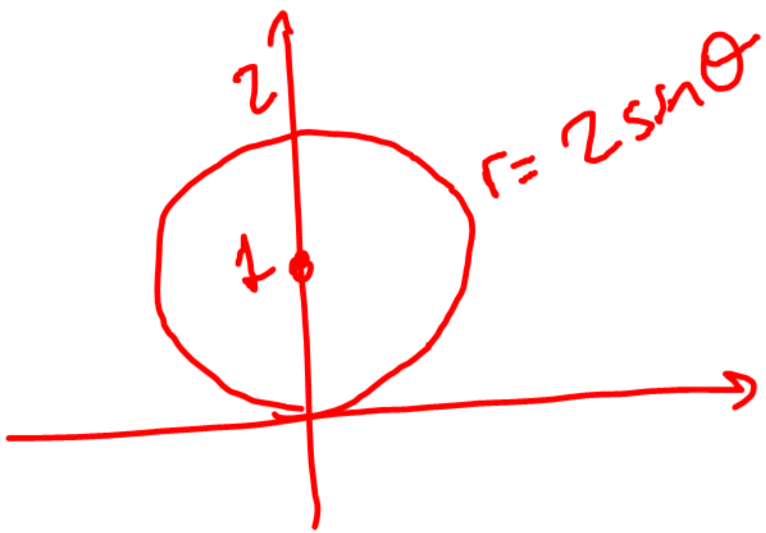
$x = r\cos\theta = 2\sin\theta\cos\theta = \sin 2\theta$

$y = r\sin\theta = 2\sin\theta\sin\theta = 2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2 2\theta + \cos^2 2\theta = 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

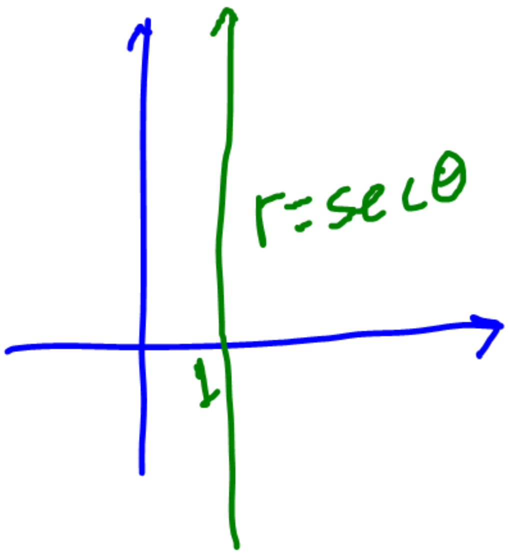
$x^2 + (1-y)^2 = 1$ circle of radius 1 centered at $(0, 1)$



2) $r = \sec\theta = \frac{1}{\cos\theta}$

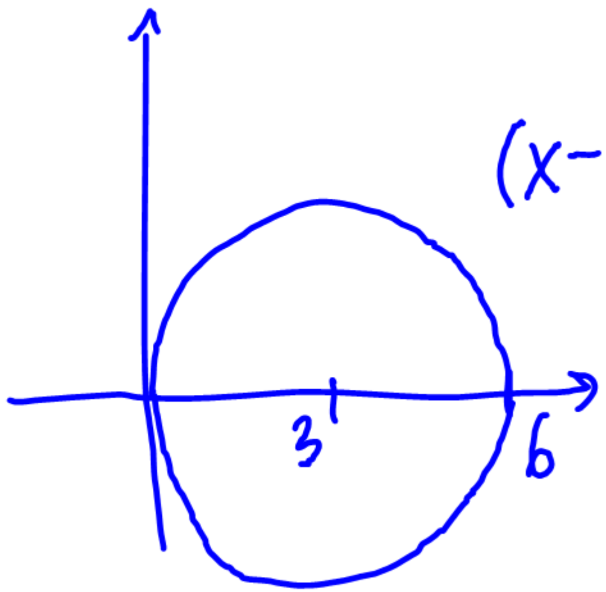
$r\cos\theta = 1$

$x = 1$



3) $r = 6\cos\theta$

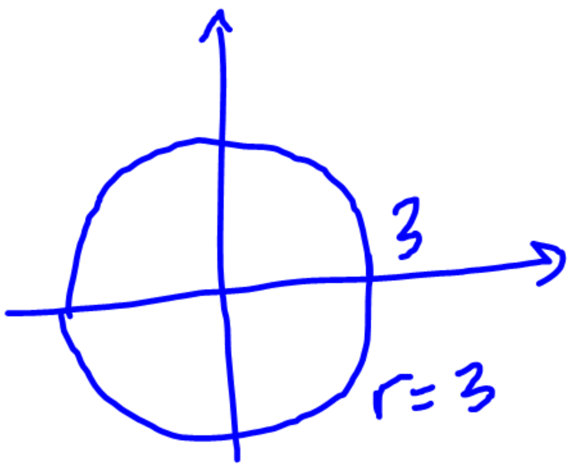
$(x-3)^2 + y^2 = 1$



4) $x^2+y^2=9$

$r^2=9$

$r=\mp 3$



5) $r = 2\tan\theta\sec\theta$

$r = 2 \cdot \frac{y}{x} \cdot \frac{1}{\cos\theta}$

$r\cos\theta = \frac{2y}{x}$

$x = \frac{2y}{x}$

$y = \frac{x^2}{2}$

parabola