Review Questions -2

- 1. Sketch the level curves z = 0, 7, 12 and 16 for $z = 16 x^2 y^2$.
- 2. Find the limit of $f(x,y) = x^3 5x^2 + xy + \cos y$ as $(x,y) \to (2,0)$.
- 3. Find the limit of $f(x, y) = \frac{y^2 xy^3}{1 x^2}$ as $(x, y) \to (1, 3)$.
- 4. Evaluate $\lim_{(x,y)\to(1,4)} e^{\sqrt{x+2y}}.$

5. Use polar coordinates to evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$.

6. Find the set of points where the function $f(x, y) = \frac{y^2 - x^2 + 1}{4 - x^2 - y^2}$ is not continuous.

- 7. Find the first order partial derivatives f_x , f_y , g_x and g_y for $f(x,y) = xy^4 2x^2y^3 + x$ and $g(x,y) = \ln\left(\sin x + e^{xy^2}\right)$.
- 8. Find the second order partial derivatives f_{xx} , f_{xy} and f_{yy} for $f(x,y) = (x^2 + y^3)^{3/2}$.
- 9. Find $f_x\left(\frac{\pi}{2}, \frac{\pi}{4}, \pi\right)$ if $f(x, y, z) = 2x + 3x \cos(z y)$.
- 10. Find $\forall f(2,1)$ if $f(a,b) = x^3 ey + x^3y^3$.
- 11. Find the gradient $\nabla f(x, y)$ if $f(x, y) = yx^2 + \ln(xy)$.
- 12. Find $\forall f(0,1)$ if $f(x,y) = e^{xy} + y^3 \sin(xy)$.
- 13. Use chain rule to determine $\frac{dz}{dt}$ where $z = \cos(yx)$, $x = t^4 2t$, $y = 1 t^3$.
- 14. Use chain rule to find the partial derivative f_u where f = f(x, y), $x = u^2 + 2v$, y = uv.
- 15. Let $f(x, y, z) = x^4 2x^2y + z^3$, $x = t^2$, $y = e^{3t}$, $z = \frac{1}{t}$. Find $\frac{df}{dt}$ by using the Chain rule.
- 16. Find the directional derivatives of the following functions in the direction of given vectors at the given points:
 - a) $f(x,y) = \sin(x-y), \ \vec{v} = \langle 1,1 \rangle, \ P = (\frac{\pi}{2}, \frac{\pi}{6}).$
 - b) $g(x, y, z) = x \ln(y + z), \ \vec{v} = 2i j + k, \ P = (2, e, e).$
- 17. Find the directional derivative of $f(x, y, z) = 2xy^2 xz^2 + yz$ at (1, 0, 2) in the direction of $\vec{v} = \langle -1, 2, 2 \rangle$.
- 18. Find the maximum rate of change of the function $f(x, y) = e^{2xy}$ at the point (0,3) and the direction in which this maximum rate of change occurs.
- 19. Find an equation of the tangent plane to the surface z = xy at (-1, 2, -2).
- 20. Find an equation of the tangent plane to the surface $z = x^2 + 4y^2$ at the point (2, 1, 8).
- 21. Find an equation for the tangent plane to the surface given by $z = \cos(x + y) + x$, at the point P = (1, -1, 0).