Abidin Kaya

/calculus 3

www.abidinkaya.wixsite.com/math

Modules

multivariable calculus

%75 attendance

1- Vectors
2- Vector valued functions
3- Differential calculus
4- Multiple integrals $\iint dxdy \iiint$
5- Vector calculus

90-100	A
80-89	B
70-79	C

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calculus 3

Parametric equations,
Polar coordinates

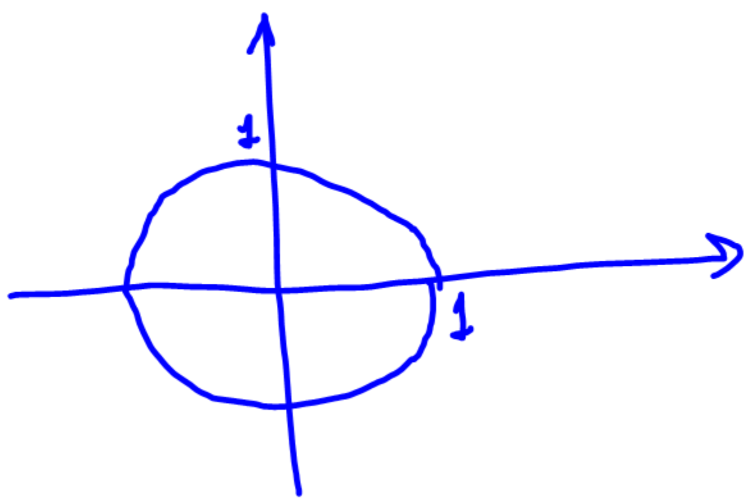
For the following exercises, sketch the curves below by eliminating the parameter t. Give the orientation of the curve.

t: parameter

2. $x = \cos(t)$, $y = \sin(t)$, $(0, 2\pi]$

$\sin^2 t + \cos^2 t = 1$

$y^2 + x^2 = 1$ circle of radius 1 centered at the origin.



For the following exercises, sketch the curves below by eliminating the parameter t. Give the orientation of the curve.

4. $x = 3 - t$, $y = 2t - 3$, $(1.5 \le t \le 3)$

$t = 3 - x$

$y = 2(3 - x) - 3$

$y = 6 - 2x - 3$

$y = 3 - 2x$ line

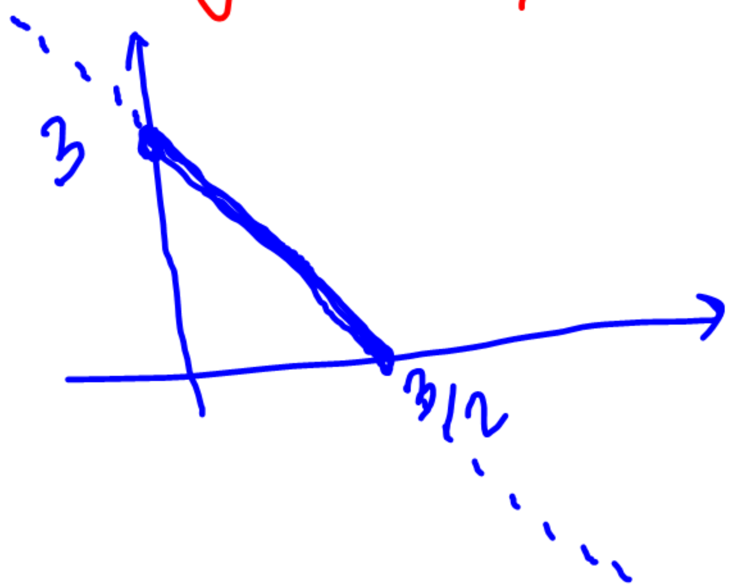
$\frac{3}{2} \le t \le 3$

$\frac{3}{2} \le 3 - x \le 3$) subtract 3

$-\frac{3}{2} \le -x \le 0$) multiply by -1

$\frac{3}{2} \ge x \ge 0$

$y = 3 - 2x, \quad 0 \le x \le \frac{3}{2}$ line segment



For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

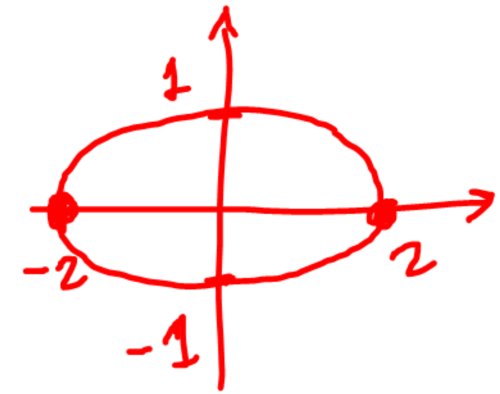
theta

parameter is θ

14. $x = 4 + 2\cos\theta, \quad y = -1 + \sin\theta$

$\frac{x-4}{2} = \cos\theta \qquad y+1 = \sin\theta$

$\frac{x^2}{4} + \frac{y^2}{1} = 1$

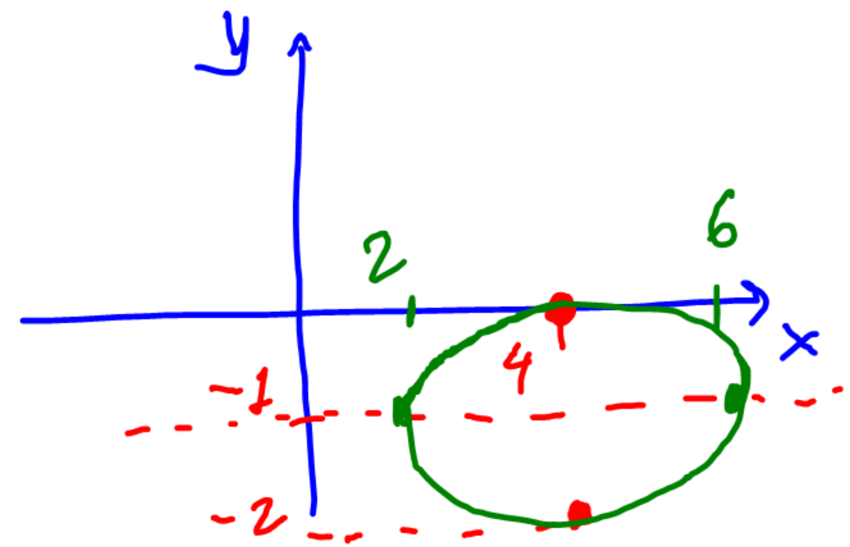


If $x=0$, $y^2=1$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x-4}{2}\right)^2 + (y+1)^2 = 1$

$\frac{(x-4)^2}{4} + (y+1)^2 = 1$ ellipse



$x=4$, $(y+1)^2 = 1$ $\quad y=0$ OR $y=-2$

$y+1=-1$

For the following exercises, convert the parametric equations of a curve into rectangular form. No sketch is necessary. State the domain of the rectangular form.

(x, y) rectangular coordinates

26. $x = t^2, \quad y = t^3$

$x^3 = t^6 = y^2$

$x^3 = y^2$

27. $x = 1 + \cos t, \quad y = 3 - \sin t$

$x - 1 = \cos t \qquad 3 - y = \sin t$

$(x-1)^2 + (3-y)^2 = 1$ circle

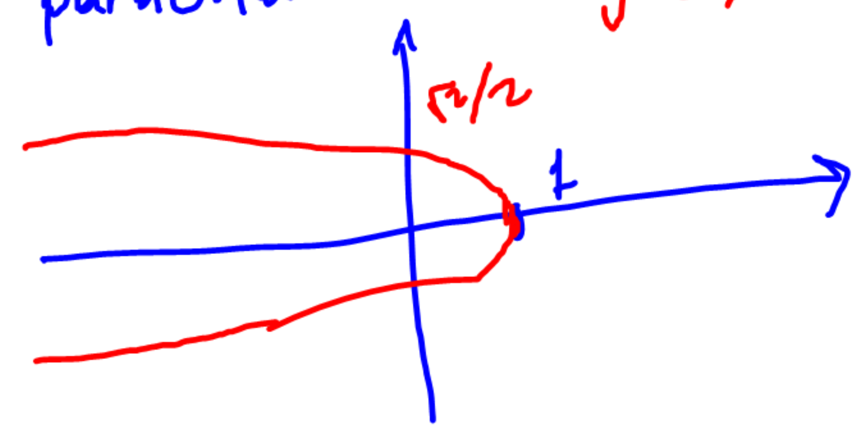
circle of radius 1 centered at $(1, 3)$

31. $x = \cos(2t), \quad y = \sin t$

$x = 1 - 2y^2$ parabola

$y = 0, \ x = 1$

$$\begin{aligned} \cos 2t &= \cos^2 t - \sin^2 t \\ &= 2\cos^2 t - 1 \\ &= 1 - 2\sin^2 t \end{aligned}$$



1.5 Find the equation of the tangent line to the curve defined by the equations

$$x(t) = t^2 - 4t, \quad y(t) = 2t^3 - 6t, \quad -2 \le t \le \cancel{3}\,6 \text{ when } t = 5.$$

Chain rule

$$m_T = \left.\frac{dy}{dx}\right|_{t=5} = \left.\frac{dy/dt}{dx/dt}\right|_{t=5} = \left.\frac{6t^2-6}{2t-4}\right|_{t=5} = \frac{150-6}{10-4} = \frac{144}{6} = 24$$

when $t=5$ $\quad x = 5^2 - 20 = 5$ and $y = 2 \times 5^3 - 6 \times 5 = 250 - 30 = 220$

the point is $(5, 220)$

$$y - 220 = 24(x - 5)$$

$\underline{y = 24x + 100}$ is the equation of the tangent.

1.6 Calculate the second derivative d^2y/dx^2 for the plane curve defined by the equations

$$x(t) = t^2 - 4t, \quad y(t) = 2t^3 - 6t, \quad -2 \le t \le 3$$

and locate any critical points on its graph.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^2-6}{2t-4}$$

quotient rule. $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{12t(2t-4)-(6t^2-6)\cdot 2}{(2t-4)^2}}{2t-4}$$

$$= \frac{12t^2-48t+12}{(2t-4)^3}$$

1.7 Find the area under the curve of the hypocycloid defined by the equations

$$x(t) = 3\cos t + \cos 3t, \quad y(t) = 3\sin t - \sin 3t, \quad 0 \le t \le \pi.$$

The area under this curve is given by

$$A = \int_a^b y(t)x'(t)\,dt.$$

$$A = \int_a^b \underbrace{f(x)}_{y}\,\underline{dx}$$

$x = x(t)$

$dx = x'(t)\,dt$

$\frac{dx}{dt} = x'(t)$

$$A = \int_0^{\pi} (3\sin t - \sin 3t) \cdot (-3\sin t - 3\sin 3t)\,dt$$

trigonometric integral

$$\int \sin^2 t\,dt = \int \left(\frac{1-\cos 2t}{2}\right) dt = \frac{t}{2} - \frac{\sin 2t}{4} + c$$

$\cos 2t = 1 - 2\sin^2 t$

$$\sin^2 t = \frac{1-\cos 2t}{2}$$

Theorem 1.3: Arc Length of a Parametric Curve

Consider the plane curve defined by the parametric equations

$$x = x(t), \quad y = y(t), \quad t_1 \le t \le t_2$$

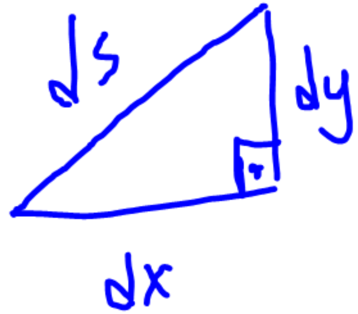
and assume that $x(t)$ and $y(t)$ are differentiable functions of t. Then the arc length of this curve is given by

$$s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

S arclength.

$y = f(x)$

$$S = \int_a^b ds$$



$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$\int_a^b ds = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

1.8 Find the arc length of the curve defined by the equations

$$x(t) = 3t^2, \quad y(t) = 2t^3, \quad 1 \le t \le 3.$$

$$s = \int_1^3 \sqrt{(6t)^2 + (6t^2)^2}\, dt = \int_1^3 \sqrt{(6t)^2(1+t^2)}\, dt$$

power rule

$$= \int_1^3 |6t| \sqrt{1+t^2}\, dt$$

$u = 1+t^2$

$du = 2t\, dt$

$$\int_2^{10} \sqrt{u} \cdot 3\, du = \frac{u^{3/2}}{3/2} \cdot 3 \Big|_2^{10}$$

$t = 1,\ u = 1+1^2 = 2$

$t = 3,\ u = 1+3^2 = 10$

$6t\, dt = 3du$

$$= 2\left(10^{3/2} - 2^{3/2}\right)$$

$$= 2\left(10\sqrt{10} - 2\sqrt{2}\right)$$